

# Minimal Model for Dilatonic Gravity and Cosmological Constant

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We consider: minimal scalar-tensor model of gravity with Brans-Dicke factor  $\omega(\Phi) \equiv 0$  and cosmological factor  $\Pi(\Phi)$ ; restrictions on it from gravitational experiments; qualitative analysis of new approach to cosmological constant problem based on the huge amount of action in Universe; determination of  $\Pi(\Phi)$  using time evolution of scale factor of Universe. PACS number(s): 04.50.+h, 04.40.Nr, 04.62.+v

## I. INTRODUCTION

The recent astrophysical observations of the type Ia supernovae [1], CMB [2], gravitational lensing and galaxies cluster's dynamics (see the review articles [3] and the references therein) gave us a strong and *independent* indications of existence of some new kind of energy in the Universe, needed to explain its accelerated expansion. Nevertheless we are not complete confident in these preliminary results, combining them with old problems of cosmology and astrophysics, we can conclude that most likely some further generalization and enlargement of the frameworks of the well established fundamental laws of physics, and in particular, of the laws of gravity, is needed.

At present, general relativity (GR) is the most successful theory of gravity at scales of laboratory-, Earth surface-, solar system- and star systems. It gives a quite good description of gravitational phenomena in the galaxies and at the scales of the whole Universe [4]. But without some essential changes it is problematic in explanation of the rotation of the galaxies and their motion in the galactic clusters, the initial singularity problem, the physics in the early Universe and the inflation [6], the present days accelerated expansion of the Universe [1]-[3] and the famous vacuum energy problem [5].

The most promising modern theories of gravity, like supergravity and (super)string theories [7], having a deep theoretical basis, incorporate naturally GR. Unfortunately, at least at present, they are not developed enough to allow a *real* experimental test, and introduce a large number of new fields without any *direct experimental evidences* for doing this.

Therefore it seems meaningful to look for some *minimal extension* of GR which is compatible with known gravitational experiments, promises to overcome at least some of the above problems and may be considered as a necessary part of more general modern theories.

It is most likely that such minimal extension must include one new scalar-field degree of freedom. Its contribution to the action of the theory can be described in different (sometimes equivalent) ways, being not fixed a priori and there exist many attempts in this direction, starting from Jordan-Fierz-Brans-Dicke theory of variable gravitational constant and its further generalizations – the so called scalar tensor theories of gravity [8] which have been proved to be the most natural extension of GR [9]. Different models of this type were used in the inflationary scenario [6] and in the more recent quintessence models [10]. For the latest development of the scalar-tensor theories with respect to accelerated expansion of the Universe one can consult the recent article [11].

One more model called a *minimal dilatonic gravity* (MDG) was proposed in the article [12]. Being a model with one additional scalar field  $\Phi$  which couples non-minimally with the space-time metric, it differs from the known inflationary models with spin zero inflation field. At the same time being mathematically equivalent to some of the quintessence models, MDG describes a *completely different physics* (see [11]), because of the relatively big mass  $m_\Phi \geq 10^{-4}eV$  of the scalar field  $\Phi$  [12]. One has to remind the reader that in the standard quintessence models two different possibilities were used until now (see [13] and the references therein): a scalar field with a typically extremely small mass  $\sim 10^{-33}eV$ , or massless scalar field with inverse-degree-potential [10]. In both cases these models suffer of some difficulties [14].

The dilatonic-gravity action in the MDG model has the form

$$\mathcal{A}_{G,\Lambda} = -\frac{c}{2\kappa} \int d^4x \sqrt{|g|} \Phi (R + 2\Lambda \Pi(\Phi)) \quad (1)$$

and corresponds to the specific choice  $\omega(\Phi) \equiv 0$  of the Brans-Dicke parameter. In the Eq. (1)  $\Lambda$  is the cosmological constant and the function  $\Pi(\Phi)$  presents a dimensionless cosmological factor.

The matter action  $\mathcal{A}_M$  and matter equations of motion are supposed to have usual GR form and do not include the scalar field  $\Phi$ . This is our *most important physical assumption* and it means that the dilaton field  $\Phi$  *does not interact directly* with the usual matter of any kind. Its influence on this matter is indirect – only due to the interaction of dilaton field  $\Phi$  with the space-time metric.

Equations for metric  $g_{\alpha\beta}$  and dilaton field  $\Phi$ :

$$\begin{aligned} \Phi (G_{\alpha\beta} - \Lambda \Pi(\Phi) g_{\alpha\beta}) - (\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square) \Phi &= \frac{\kappa}{c^2} T_{\alpha\beta}, \\ \square \Phi + \Lambda \frac{dV}{d\Phi}(\Phi) &= \frac{\kappa}{3c^2} T \end{aligned} \quad (2)$$

yield usual energy-momentum conservation law

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$$\nabla_\alpha T_\beta^\alpha = 0 \quad (3)$$

and the important relation:

$$R + 2\Lambda \frac{dU}{d\Phi}(\Phi) = 0. \quad (4)$$

The quantities  $\frac{dV}{d\Phi} := \frac{2}{3}\Phi(\frac{d\Pi}{d\Phi} - \Pi)$  and  $U(\Phi) := \Phi\Pi(\Phi)$  introduce *dilatonic potential*  $V(\Phi)$  – in Eq.(2) and *cosmological potential*  $U(\Phi)$  – in Eq.(4).

It is remarkable that introducing the dilaton field  $\Phi$  in MDG we do not need to prescribe some new charges, or other novel properties to the usual matter. As seen from Eq. (2), this field is completely determined by the trace of the energy-momentum tensor of matter.

One has to stress that because of the condition  $\omega(\Phi) \equiv 0$  the existence of nontrivial relativistic dynamics and propagation of the dilaton field  $\Phi$  in vacuum is deeply connected with *the nonzero space-time curvature*. Technically, the second order dynamical equation for  $\Phi$  in the system (2) is obtained by contraction of the generalized Einstein equations and making use of the algebraic relation (4), derived by variation of the action (1) with respect to the dilaton field. The second order derivatives of the field  $\Phi$  are created during the two-fold integration by parts of the corresponding terms in variation of the specific action (1) with respect to the metric, not with respect to the very field  $\Phi$ . Therefore, in a flat space-time, the zero scalar curvature  $R$  in action (1) would lead to a non-existence of dynamics of the dilaton field. This simple argument, *being specific for MDG*, shows a deep connection of the field  $\Phi$  with the space-time curvature, i.e., with the gravity. Therefore it seems natural to treat the dilaton field  $\Phi$  as a "scalar part of gravity", instead of considering it as a new kind of matter scalar field.

The action (1) can be considered as a Helmholtz action of nonlinear gravity (see [15] and the references therein) with lagrangian  $L_{NLG} \sim \sqrt{|g|}f(r)$ ,  $r = R/\Lambda$  being dimensionless scalar curvature [12]. Indeed, because of the absence of a Brans-Dicke kinetic term  $\sim \omega(\Phi)(\nabla\Phi)^2$ , the variation of the action (1) with respect to the dilaton field  $\Phi$  gives the local *algebraic* relation (4) between scalar curvature  $R$  and dilaton  $\Phi$ , instead of differential equation. If  $\frac{d^2}{d\Phi^2}U(\Phi) \neq 0$ , one can solve the relation (4) with respect to the field  $\Phi$  and this field becomes a *local* function of the scalar curvature:  $\Phi = \Phi(r)$ . (For  $\omega \neq 0$  the last relation would have a non-local integral form.) The substitution of this function back into the action (1) transforms it to the action of nonlinear theories of gravity  $\mathcal{A}_{NLG} = -\frac{c\Lambda}{2\kappa} \int d^4x \sqrt{|g|} f(r)$  with  $f(r) = r\Phi(r) + 2U(\Phi(r))$ . The inverse correspondence – from NLG to MDG – can be described in a simple way, too. For a given function  $f(r)$  we have to solve the algebraic equation  $\frac{d}{dr}f(r) = \Phi$  with respect to the variable  $r$ . This gives a function  $r = r(\Phi)$ . Then  $U(\Phi) = -\int r(\Phi)d\Phi + const$ .

It is well known that the nonlinear gravity can be created by the quantum corrections which appear after quantization of the classical fields in curved space-time

[16]. This was the physical basis of the original Starobinsky model of inflation [17]. The modern development of this model one can find in the recent articles [18].

The total energy momentum of the vacuum fluctuations in this model cannot be obtained by varying a local action, see the article by A. Vilenkin in [17]. Therefore, in the general case MDG differs both physically and mathematically from the Starobinsky model and its modern developments. For example, the cosmological perturbations in the two models are essentially different. Nevertheless, it turns out that under proper particular choice of the cosmological factor  $\Pi(\Phi)$ , MDG coincides with Starobinsky model in the case of *the conformally flat* Robertson-Walker (RW) metric. This happens just because the term which yields the essential difference between the two models is proportional to the Weyl conformal curvature tensor, but this curvature is zero in the conformally flat spaces with RW metric [19].

The last consideration indicates that probably we have to look for the roots of our MDG in quantum field theory in curved space-time.

At present, a well known candidate for a self-consistent quantum theory of gravity is the superstring theory. It turns out that the MDG can be considered as a four dimensional version of the low energy limit (LEL) of the string theory in some *new frame*. In this new frame the action (1) is precisely the stringy three-level effective action *for the metric and the dilaton only*, corresponding to the lowest order of string loop expansion, i.e., for the most constant part of the string theory which has the same form in all string models.

Indeed, let us consider the  $D$ -dimensional LEL lagrangian in stringy frame (SF) [7]:

$${}_S L_{LEL} \sim \sqrt{|{}_S g|} e^{-2\phi} ({}_S R + 4{}_S (\partial\phi)^2 + {}_S V_{SUSY}(\phi)).$$

After Weyl conformal transformation:  $g_{\mu\nu} \rightarrow e^{b\phi} g_{\mu\nu}$  to some new conformal frame, which depends on the parameter  $b$ , it acquires the form:

$$L_{LEL} \sim \sqrt{|g|} e^{p_1(b)\phi} (R + p_2(b)(\partial\phi)^2 + V_{SUSY}(\phi))$$

where  $p_1(b) = (D-2)b-2$  and  $p_2(b) = 4-4(D-1)b+(D-1)(D-2)b^2$  are polynomials in  $b$  of degree 1 and 2, correspondingly.

Now, if we chose the parameter  $b = b_E = \frac{2}{D-2}$ , i.e., if  $p_1(b_E) = 0$ , we will obtain the well known Einstein frame (EF) in which

$${}_E L_{LEL} \sim \sqrt{|{}_E g|} \left( {}_E R + \frac{4}{D-2} {}_E (\partial\phi)^2 + {}_E V_{SUSY}(\phi) \right). \quad (5)$$

But there exists another simple choice:  $p_2(b_F) = 0$  which gives  $b = b_F = \frac{2}{D-2} \left( 1 \pm \frac{1}{\sqrt{D-1}} \right)$  and a new conformal frame which we call a *fundamental frame* (FF). If one sets  $\Phi := \exp\left(\pm \frac{2\phi}{\sqrt{D-1}}\right)$ , one reaches the following simple form of string LEL lagrangian for metric and dilaton fields in FF:

$${}_F L_{LEL} \sim \sqrt{|{}_F g|} (\Phi {}_F R + {}_F U_{SUSY}(\Phi)). \quad (6)$$

Obviously, under proper re-interpretation of all terms in it, this lagrangian gives the action (1) as a truncated 4-dimensional LEL action in graviton-dilaton sector of the string theory, i.e., as an action, obtained neglecting i) the contribution of all other string excitations, ii) the contribution of the fields, connected with the higher dimensions and iii) contribution of higher order terms of string perturbation theory.

At present the exact form of the potential  ${}_S V_{SUSY}(\phi)$  is not known. It may originate from SUSY breaking due to gaugino condensation [7], or may appear in the theory in some more sophisticated way.

The form of lagrangian (6) in FF is more simple than the corresponding form in EF – (5). In the FF we have a more direct interpretation of the MDG in the spirit of Jordan-Fierz-Brans-Dicke theory, i.e. as a theory with variable gravitational "constant"  $G = \bar{G}\Phi^{-1} = \bar{G} \exp\left(\mp \frac{2\phi}{\sqrt{D-1}}\right)$ . Because of the quadratic equation  $p_2(b_F) = 0$  we have two possible values of the parameter  $b_F$  and two fundamental frames:  $FF_{\pm}$ . In each of them the gravitational "constant" depends exponentially on the original (i.e., defined in the SF) dilaton field  $\phi$ , but the signs of the arguments in the corresponding exponents are opposite.

Note that using other values of the parameter  $b$  in the exponential factor  $e^{b\phi}$  of the Weyl conformal transformation, one can produce a kinetic term for the dilaton in the action of theory with any desired value of the coefficient  $\omega$ . It is clear that among the infinitely many possible conformal frames the above two: the Einstein frame and the fundamental frame are distinguished ones.

Usually one prefers to work in Einstein frame, because in it the field variables  ${}_E g_{\mu\nu}$  and  $\phi$  are separated and the corresponding Cauchy problem is well posed [11]. Thus, the choice of Einstein frame is a convenient mathematical tool which is analogous to the choice of normal coordinates in usual mechanics and field theory. In the reference [11] an additional description of this property of Einstein frame is stressed: in it we have no mathematical mixing between "true" helicity-0 excitation  $\phi$  and helicity-2 excitation  ${}_E g_{\mu\nu}$ , in contrast to the situation in other conformal frames. For example, in Jordan frame for scalar-tensor theories of general type these excitation are mixed, see for details the reference [11].

It is well known that the EF is not the physical one and one needs to find the physical frame to reach a right interpretation of the results, see for example [11] and the references therein.

In MDG the true separation of the *physical* properties of the helicity-0 and helicity-1 degrees of freedom takes place in the fundamental frame:

1) Because of the condition  $\omega(\Phi) \equiv 0$  in MDG we have a local functional dependence between the fields  $\phi$  and  $\Phi := \exp\left(\pm \frac{2\phi}{\sqrt{D-1}}\right)$ , instead of the differential equation (2.4b) in the reference [11]. Therefore the field  $\Phi$  is not

*physically* different from the field  $\phi$ , at least locally, and it carries all physical properties of the true helicity-0 degree of freedom, nevertheless these properties are described in a different mathematical way.

2) In contrast to other frames in FF the helicity-0 field degree of freedom does not interact directly with matter at all, and only the helicity-2 field degree of freedom is responsible for the interaction of gravity with matter.

Some time ago, the FF was recognized to be an useful tool in the two-dimensional models of dilatonic gravity, both classical and quantum ones – see the recent article [20] and the references therein. Moreover, for the exact quantization of all  $D = 2$  models of pure dilatonic gravity *with arbitrary potential*  $U(\Phi)$  it turned out to be critical to work just in the FF frame [21].

Now our main physical assumption may be formulated as a hypothesis that *the week equivalence principle is valid precisely in FF*. This means that just in FF we are to set the action for usual matter in its GR form, i.e., we accept the FF as a physical frame in which the physical observations and experiments are performed. They are described in the terms of standard non-Euclidean 4-space-time geometry. In other words we assume that in these experiments one is testing just the FF geometry using *the real physical objects*. Hence the name "fundamental frame".

The string theory is supposed to work in the well studied and relatively simple way at Planck scales, but in it one needs some additional (at present unknown) procedures for describing the usual matter. As a result, at present we do not know how to treat the real matter (build of electrons, protons,  $\pi$ -mesons, e.t.c.) in the framework of string theory. In particular, the interaction of stringy dilaton with usual matter is unknown, see the discussion of this problem and possible violation of the week equivalence principle due to the interaction of dilaton with usual matter in [23]. In this situation our choice of FF as a physical frame justifies the very string theory in the spirit of Fierz article [22], where he first noticed that the extremely high precision of the week equivalence principle (nowadays it is at the level of  $10^{-12}$ ) suggests that the coupling of matter and gravity must have *an exact metric form*, but there still exist an open possibilities to change the Einstein-Hilbert action of GR, see the recent reference [24], too. If successful, our model of MDG can help the further development of string theory as a possible physical description of the real world.

The action (1) appears, too, in a new model of gravity with torsion and unusual local conformal symmetry after its breaking in metric-dilaton sector [26] and in  $D = 5$  Kaluza-Klein theories [27].

For boson stars the MDG was tested in [28]. There it was shown that the star structure is slightly sensitive mainly to the mass term in the dilatonic potential (typically into a few percent) and do not depend on the exact form of this potential.

Investigation of MDG was started by O'Hanlon in connection with Fujii's theory of massive dilaton [25], but

without any relation with cosmological constant problem and other problems of cosmology and astrophysics.

An *essential new element* of our MDG is the *nonzero cosmological constant*  $\Lambda$  [12]. Nevertheless at present still exist doubts in astrophysical data:  $\Omega_\Lambda = .65 \pm .13$ ,  $H_0 = (65 \pm 5) \text{ km s}^{-1} \text{ Mpc}^{-1}$  which determine

$$\Lambda^{obs} = 3\Omega_\Lambda H_0^2 c^{-2} = (.98 \pm .34) \times 10^{-56} \text{ cm}^{-2}$$

we accept this observed value of cosmological constant *as a basic quantity* which *defines natural units* for all other cosmological quantities, namely: cosmological length:  $A_c := 1/\sqrt{\Lambda^{obs}} = (1.02 \pm .18) \times 10^{28} \text{ cm}$ , cosmological time:  $T_c := A_c/c = (3.4 \pm .6) \times 10^{17} \text{ s} = (10.8 \pm 1.9) \text{ Gyr}$ , cosmological energy density:  $\varepsilon_c := \frac{\Lambda c^2}{\kappa} = (1.16 \pm .41) \times 10^{-7} \text{ g cm}^{-1} \text{ s}^{-2}$ , cosmological energy:  $E_c := 3A_c^3 \varepsilon_c = 3\Lambda^{-1/2} c^2 \kappa^{-1} = (3.7 \pm .7) \times 10^{77} \text{ erg}$ , cosmological momentum:  $P_c := 3c/(\kappa\sqrt{\Lambda^{obs}}) = (1.2 \pm .2) \times 10^{67} \text{ g cm s}^{-1}$  and *cosmological unit for action*:

$$\mathcal{A}_c := 3c/(\kappa\Lambda^{obs}) = (1.2 \pm .4) \times 10^{122} \hbar,$$

$\kappa$  being Einstein constant. Further we use dimensionless variables like:  $\tau := t/T_c$ ,  $a := A/A_c$ ,  $h := H T_c$  ( $H := A^{-1} dA/dt$  being Hubble parameter),  $\epsilon_c := \varepsilon_c/|\varepsilon_c| = \pm 1$ ,  $\epsilon := \varepsilon/|\varepsilon_c|$ -matter energy density, etc.

In our special scalar-tensor model of gravity the cosmological factor  $\Pi(\Phi)$  is the *only unknown function* which has to be chosen to comply with gravitational experiments and observations and to solve the *inverse cosmological problem* described in the last Section.

## II. SOLAR SYSTEM AND EARTH-SURFACE GRAVITATIONAL EXPERIMENTS

From known gravitational experiments one can derive the following properties of cosmological factor  $\Pi(\Phi)$ :

1. MDG with  $\Lambda = 0$  contradicts to solar system gravitational experiments. The cosmological term  $\Lambda \Pi(\Phi) \neq 0$  in action (1) is needed to overcome this problem.

2. In contrast to O'Hanlon's model we wish MDG to reproduce GR with  $\Lambda \neq 0$  for some  $\Phi = \bar{\Phi} = \text{const} \neq 0$ , i.e., the original de Sitter solution. Then we derive for cosmological factor of this solution the conditions:

$$\Pi(\bar{\Phi}) = 1, \quad \frac{d\Pi}{d\Phi}(\bar{\Phi}) = \bar{\Phi}^{-1}, \quad \frac{d^2\Pi}{d\Phi^2}(\bar{\Phi}) = \frac{3}{2}p^{-2}\bar{\Phi}^{-2} \quad (7)$$

as follows: i) From action (1) we obtain the first *normalization condition* and Einstein constant  $\kappa = \bar{\kappa}/\bar{\Phi}$ ; ii) In vacuum, far from matter MDG have to allow *weak field approximation*:  $\Phi = \bar{\Phi}(1 + \zeta)$  ( $|\zeta| \ll 1$ ). Then the *linearized* dilaton equation (2):  $\square\zeta + \zeta/l_\Phi^2 = \frac{\kappa}{3c^2}T$  gives the second condition and iii) Taylor series expansion of the function  $\frac{d\Pi}{d\Phi}(\Phi)$  around the value  $\bar{\Phi}$  introduces dimensionless Compton length of dilaton  $p = \frac{l_\Phi}{A_c}$  and gives the third of conditions (7). As a result we obtain

$$\Pi = 1 + \zeta + \frac{3}{4p^2}\zeta^2 + O(\zeta^3). \quad (8)$$

3. *Point particles of masses*  $m_a$  as source of metric and dilaton fields give in Newtonian approximation gravitational potential  $\varphi(r)$  and dilaton field  $\Phi(r)$ :

$$\begin{aligned} \varphi(r)/c^2 = & -\frac{G}{c^2} \sum_a \frac{m_a}{|r-r_a|} \left(1 + \alpha(p) e^{-|r-r_a|/l_\Phi}\right) \\ & - \frac{1}{6} p^2 \sum_a \frac{m_a}{M} (|r-r_a|/l_\Phi)^2, \end{aligned} \quad (9)$$

$$\Phi(r)/\bar{\Phi} = 1 + \frac{2}{3} \frac{G}{c^2(1-\frac{4}{3}p^2)} \sum_a \frac{m_a}{|r-r_a|} e^{-|r-r_a|/l_\Phi}, \quad (10)$$

$G = \frac{\kappa c^2}{8\pi}(1-\frac{4}{3}p^2)$  is Newton constant,  $M = \sum_a m_a$ . The term  $-\frac{1}{6}p^2 \sum_a \frac{m_a}{M} (|r-r_a|/l_\Phi)^2 = -\frac{1}{6}\Lambda|r - \sum_a \frac{m_a}{M}r_a|^2 + \text{const}$  in  $\varphi$  is known from GR with  $\Lambda \neq 0$ . It represents an *universal anti-gravitational interaction* of test mass  $m$  with any mass  $m_a$  via repellent elastic force

$$F_{\Lambda a} = \frac{1}{3}\Lambda m c^2 \frac{m_a}{M} (r - r_a). \quad (11)$$

For *solar system distances*  $l \leq 1000 \text{ AU}$  neglecting the  $\Lambda$  term (of order  $\leq 10^{-24}$ ) we compare the gravitational potential  $\varphi$  with specific MDG coefficient  $\alpha(p) = \frac{1+4p^2}{3-4p^2}$  with Cavendish type experiments and obtain an experimental constraint  $l_\Phi \leq 1.6 \text{ [mm]}$ , or  $p < 2 \times 10^{-29}$ . Hence, in the solar system the factor  $e^{-l/l_\Phi}$  has fantastic small values ( $< \exp(-10^{13})$  for the Earth-Sun distances  $l$ , or  $< \exp(-3 \times 10^{10})$  for the Earth-Moon distances  $l$ ) and *there is no hope to find some differences between MDG and GR* in this domain.

The corresponding constraint  $m_\Phi c^2 \geq 10^{-4} [\text{eV}]$  does not exclude a small value (with respect to the elementary particles scales) for the rest energy of hypothetical  $\Phi$ -particle.

5. The *parameterized-post-Newtonian (PPN) solution* of equation (2) is complicated, but because of the constraint  $p < 10^{-28}$  we may use with great precision Helbig's PPN formalism [29] (for  $\alpha = \frac{1}{3}$ ). Because of the condition  $\omega \equiv 0$  we obtain much more definite predictions than usual general relations between  $\alpha$  and the length  $l_\Phi$ :

• *Nordtvedt Effect*:

In MDG a body with significant gravitational self-energy  $E_G = \sum_{b \neq c} G \frac{m_b m_c}{|r_b - r_c|}$  will not move along geodesics due to *additional universal anti-gravitational force*:

$$F_N = -\frac{2}{3} E_G \nabla \Phi. \quad (12)$$

For usual bodies it is too small even at distances  $|r - r_a| \leq l_\Phi$ , because of the small factor  $E_G$ . Hence, in MDG we have no strict strong equivalence principle nevertheless *the weak equivalence principle is not violated*.

The experimental data for Nordtvedt effect, caused by the Sun, are formulated as a constraint  $\eta = 0 \pm .0015$  on the parameter  $\eta$  which in MDG becomes a function of

the distance  $l$  to the source:  $\eta(l) = -\frac{1}{2}(1 + l/l_\Phi)e^{-l/l_\Phi}$ . This gives constraint  $l_\Phi \leq 2 \times 10^{10}[m]$ .

- *Time Delay of Electromagnetic Waves*

The action of electromagnetic field does not depend on the field  $\Phi$ . Therefore influence of  $\Phi$  on the electromagnetic waves in vacuum is possible only via influence of  $\Phi$  on the space-time metric. The solar system measurements of the time delay of the electromagnetic pulses give the value  $\gamma = 1 \pm .001$  of this post Newtonian parameter. In MDG this yields the relation  $(1 \pm .001)g(l_{AU}) = 1$  and gives once more the constraint  $l_\Phi \leq 2 \times 10^{10}[m]$ . Here  $g(l) := 1 + \frac{1}{3}(1 + l/l_\Phi)e^{-l/l_\Phi}$ .

- *Perihelion Shift*

For the perihelion shift of a planet orbiting around the Sun (with mass  $M_\odot$ ) in MDG we have:  $\delta\varphi = \frac{k(l_p)}{g(l_p)}\delta\varphi_{GR}$ . Here  $l_p$  is the semimajor axis of the orbit of planet and  $k(l_p) \approx 1 + \frac{1}{18}\left(4 + \frac{l_p^2}{l_\Phi^2} \frac{l_p c^2}{GM_\odot}\right)e^{-l_p/l_\Phi} - \frac{1}{27}e^{-2l_p/l_\Phi}$  is obtained neglecting its eccentricity. The observed value of perihelion shift of Mercury gives the constraint  $l_\Phi \leq 10^9[m]$ .

Hence, the known data show that dilaton field  $\Phi$  does not cause observable deviations from GR in solar system. Essential deviations from Newton law of gravity may not be expected at distances greater than few *mm*.

### III. VACUUM ENERGY AND TRUE VACUUM SOLUTION IN MDG

Consider the total (true) tensor of energy momentum:

$$TT_{\mu\nu} := T_{\mu\nu} + \langle \rho_0 \rangle c^2 g_{\mu\nu}, \quad (13)$$

$\langle \rho_0 \rangle$  being the averaged energy density of the zero quantum fluctuations. For *true vacuum solution* of MDG:  $\Phi = \Phi_0 = \text{const}$ ,  $g_{\mu\nu} = \eta_{\mu\nu}$  (Minkowski metric) from field equations (2) we obtain:

$$\Phi_0 \frac{d\Pi}{d\Pi}(\Phi_0) + \Pi(\Phi_0) = 0 \quad (14)$$

$$T_{\mu\nu}^0 = -\frac{c^2}{\kappa} \Lambda U_0 \eta_{\mu\nu} = TT_{\mu\nu}^0 - \langle \rho_0 \rangle c^2 \eta_{\mu\nu}, \quad (15)$$

where  $U_0 = \Phi_0 \Pi(\Phi_0) = \Phi_0 \Pi_0$ . But for true vacuum solution we must have  $TT_{\mu\nu}^0 \equiv 0$  and then

$$\langle \rho_0 \rangle = \kappa^{-1} \Lambda U_0 = \kappa^{-1} \Lambda \Pi_0. \quad (16)$$

Hence, true vacuum ( $TT_{\mu\nu} = 0$ ) yields Minkowski space-time, but physical vacuum ( $T_{\mu\nu} = 0$ ) yields de Sitter space-time. In our model the real world is de Sitter Universe created by zero quantum vacuum fluctuations and perturbed by other matter and radiation fields.

For  $\langle \rho_0 \rangle$  calculated using Plank length as a quantum cutoff the observed value of  $\Lambda$  gives:

$$\kappa < \rho_0 > / \Lambda = U(\Phi_0)/U(\bar{\Phi}) \approx 10^{122}. \quad (17)$$

This huge number yields the famous *cosmological constant problem* and varies from  $10^{118}$  to  $10^{123}$  in different articles [5]. We see that: 1) It is obviously close in order to the ratio of cosmological action  $\mathcal{A}_c$  and Planck constant  $\hbar$ :  $U(\Phi_0)/U(\bar{\Phi}) \approx \mathcal{A}_c/\hbar$ ; 2) in MDG there is no crisis caused by this number, because it gives the ratio of the values of cosmological potential for different solutions:  $\Phi_0$  and  $\bar{\Phi}$ , i.e. *for different states of the Universe*.

If we calculate the values  $\mathcal{A}_{G,\Lambda}^0$  and  $\bar{\mathcal{A}}_{G,\Lambda}$  of the very action (1) and introduce corresponding specific actions per unit volume:  $\alpha_0 = -\Lambda c \bar{\kappa}^{-1} U_0$  and  $\bar{\alpha} = \Lambda c \bar{\kappa}^{-1} \bar{U}$ , we can rewrite the above observed result in the form  $\bar{\alpha} \approx -\alpha_0 \times \hbar/\mathcal{A}_c = |\alpha_0| \times 10^{-122}$ .

We hope that this new formulation of cosmological constant problem will bring us to its resolution.

It's natural to think that the huge ratio  $\mathcal{A}_c/\hbar \approx 10^{122}$  is produced during the evolution of the Universe. To perform qualitative analysis we consider first the simplest model of Universe build of Bohr hydrogen atoms in ground state, i.e. we describe the whole content of the Universe using such *effective Bohr hydrogen* (EBH) atoms. Then for the time of the existence of Universe  $T_U \sim 4 \times 10^{17} \text{sec}$  one EBH atom with Bohr angular velocity  $\omega_B = m_e e^4 \hbar^{-3} \sim 4 \times 10^{16} \text{sec}^{-1}$  accumulates classical action  $\mathcal{A}_{EBH} = 3/2 \omega_B T_U \hbar \sim 2.4 \times 10^{34} \hbar$ . Hence, the number of EBH in Universe, needed to explain the present day action  $\sim \mathcal{A}_c$ , must be  $N_{EBH} \sim 5 \times 10^{87}$ . This seems to be quite reasonable number, taking into account that the observed number of baryons in Universe is  $N_{baryons}^{obs} \sim 10^{78}$  and we see that in our approach we have disposable some 9 orders of magnitude to solve cosmological constant problem taking into account the contribution of all other constituents of matter and radiation (quarks, leptons, gamma quanta, etc) during the evolution of Universe from the Big Bang to the present epoch. Neglecting the temperature evolution of the Universe, we obtain an accumulated action  $\mathcal{A}_\gamma \sim \omega_\gamma T_U \hbar \sim 10^{30} \hbar$  for one  $\gamma$ -quanta of CMB (which is most significant part of radiation in Universe). A simple estimate for Bohr-like angular velocity of constituent quarks in proton is  $\omega_{Bq} = m_e/m_q (r_B/r_p)^2 \omega_B \approx 10^7 \omega_B$  (for mass of constituent quark  $m_q \sim 5 \text{MeV}$ , Bohr radius  $r_B$  and radius of proton  $r_p \sim 8 \times 10^{-13} \text{cm}$ ). Then the action, accumulated by constituent quarks in one proton during evolution of the Universe, is  $\mathcal{A}_p \sim \omega_{Bq} T_U \hbar \sim 10^{42} \hbar$ . This gives unexpectedly good estimation for the number of effective protons (ep) in the Universe:  $N_{ep} \sim 10^{80}$ . We may use the left-off two orders of magnitude to take into account contribution of the other matter constituents and of the temperature evolution of the Universe: during the short-time initial hot phase some additional action must be produced.

The main conclusion of this qualitative consideration based on classical mechanics and simplest application of basic quantum relations is that actually in MDG *the observed nonzero value* of cosmological constant  $\Lambda^{obs} \neq 0$  *restricts the number of degrees of freedom* in the observ-

able Universe and maybe forbids the existence of more deep levels of matter below the quark level.

#### IV. APPLICATION OF MDG IN COSMOLOGY

In MDG for Freedman-Robertson-Walker (FRW) adiabatic homogeneous isotropic Universe with  $ds_{FRW}^2 = c^2 dt^2 - A^2 dl_k^2$ ,  $t = T_c \tau$ ,  $A(t) = A_c a(\tau)$  and dimensionless  $dl_k^2 = \frac{dl^2}{1-kl^2} + l^2(d\theta^2 + \sin^2\theta d\varphi^2)$  ( $k = -1, 0, 1$ ) in presence of matter with energy-density  $\varepsilon = \varepsilon_c \epsilon(a)/\bar{\Phi}$  and pressure  $p = \varepsilon_c p_\epsilon(a)/\bar{\Phi}$  dynamical equations are:

$$\frac{1}{a} \frac{d^2 a}{d\tau^2} + \frac{1}{a^2} \left( \frac{da}{d\tau} \right)^2 + \frac{k}{a^2} = \frac{1}{3} \left( \Phi \frac{d\Pi}{d\Phi}(\Phi) + \Pi(\Phi) \right),$$

$$\frac{1}{a} \frac{da}{d\tau} \frac{d\Phi}{d\tau} + \Phi \left( \frac{1}{a^2} \left( \frac{da}{d\tau} \right)^2 + \frac{k}{a^2} \right) = \frac{1}{3} \left( \Phi \Pi(\Phi) + \epsilon(a) \right). \quad (18)$$

The use of Hubble parameter  $h(a) = a^{-1} \frac{da}{d\tau}(\tau(a))$  (where  $\tau(a)$  is the inverse function of  $a(\tau)$ ), new variable  $\lambda = \ln a$  and prime for differentiation with respect to  $\lambda$  gives the system for  $\Phi(\lambda)$  and  $h^2(\lambda)$ :

$$\frac{1}{2} (h^2)' + 2h^2 + ke^{-2\lambda} = \frac{1}{3} \left( \Phi \frac{d\Pi}{d\Phi}(\Phi) + \Pi(\Phi) \right),$$

$$h^2 \Phi' + (h^2 + ke^{-2\lambda}) \Phi = \frac{1}{3} (\Phi \Pi(\Phi) + \epsilon(e^\lambda))$$

and relation  $\tau(a) = \int_{a_{in}}^a da / (a h(a)) + \tau_{in}$ . Excluding cosmological factor  $\Pi(\Phi)$  we obtain the equation:

$$\Phi'' + \left( \frac{h'}{h} - 1 \right) \Phi' + 2 \left( \frac{h'}{h} - k h^{-2} e^{-2\lambda} \right) \Phi = \frac{1}{3 h^2} \epsilon', \quad (19)$$

or in terms of the function  $\psi(a) = \sqrt{|h(a)|/a} \Phi(a)$ :

$$\psi'' + n^2 \psi = \delta, \quad (20)$$

$$-n^2 = \frac{1}{2} \frac{h''}{h} - \frac{1}{4} \left( \frac{h'}{h} \right)^2 - \frac{5}{2} \frac{h'}{h} + \frac{1}{4} + \frac{2k}{h^2} e^{-2\lambda}, \delta = \frac{1}{3} \sqrt{a/|h|}^3 \frac{d\epsilon}{da}.$$

Now we are ready to consider *the inverse cosmological problem*: to find a cosmological factor  $\Pi(\Phi)$  which yield given evolution  $a(\tau)$  of the Universe. A remarkable property of MDG is that unique solution of this problem exist for almost any three times differentiable function  $a(\tau)$ ; the values of all "bar" quantities (including  $\bar{\kappa}$  in action (1)) may be determined from time evolution  $a(\tau)$  of the Universe via the solution  $\bar{\lambda} = \ln \bar{a}$  of the Eq. (4).

Indeed: for known  $a(\tau)$  construct the function  $h(\lambda)$  and find the point  $\bar{\lambda}$  as real solution of the algebraic equation  $r(\bar{\lambda}) = -4$ ,  $r(\lambda) = -6 \left( \frac{1}{2} (h^2)' + 2h^2 + ke^{-2\lambda} \right)$  being dimensionless scalar curvature:  $r = R/\Lambda$ . Then using Eq. (7) obtain  $\bar{\Phi} = -4\bar{\epsilon} (1 + \frac{4}{3} p^2) / (\bar{j}'_{00} (1 + \frac{4}{3} p^2) + 4p^2 \bar{h}^2 \bar{r}')$ ,  $\bar{\Phi}'/\bar{\Phi} = -\frac{1}{3} p^2 \bar{r}' / (1 + \frac{4}{3} p^2)$ ;  $j_{00} = G_{00}/\Lambda = 3 (h^2 + ke^{-2\lambda})$  is dimensionless 00-component of Einstein tensor. In their turn quantities  $\bar{\Phi}$  and  $\bar{\Phi}'$  determine the values of constants  $C_{1,2}$  in general solution  $\Phi(\lambda)$  of Eq. (19):  $\Phi(\lambda) = C_1 \Phi_1(\lambda) + C_2 \Phi_2(\lambda) + \Phi_\epsilon(\lambda)$  where  $\Phi_1(\lambda)$  and  $\Phi_2(\lambda)$  are a fundamental system of solutions of corresponding homogeneous equation and

$$\Phi_\epsilon = \frac{\bar{a}}{(3\bar{h}\bar{\Delta})} \left( \Phi_2 \int_{\bar{\lambda}}^\lambda d\epsilon \frac{\Phi_1}{ah} - \Phi_1 \int_{\bar{\lambda}}^\lambda d\epsilon \frac{\Phi_2}{ah} \right),$$

$\Delta(\lambda) = \Phi_1 \Phi_2' - \Phi_2 \Phi_1'$ . The dependence on  $\lambda$  of cosmological factor  $\Pi$  and potential  $V$  are given by equations

$$\Pi(\lambda) = j_{00} + 3h^2 \Phi' / \Phi - \epsilon / \Phi,$$

$$V(\lambda) = \frac{2}{3} \int \Phi (\Phi \Pi' - \Phi' \Pi) d\lambda \quad (21)$$

which define functions  $\Pi(\Phi)$  and  $V(\Phi)$  implicitly.

This mathematical result shows maybe the best way to study SUSY breaking and the corresponding potential  $V_{SUSY}(\phi)$ : we have to reconstruct the *real time evolution*  $a(\tau)$  of the Universe from astrophysical observations.

Finally we stress following specific properties of MDG:

1) If  $n > 0$  *dilatonic field*  $\Phi(a)$  *oscillates*; if  $n < 0$  such oscillations do not exist. The change of sign of dilaton field  $\Phi$  yields phase transitions of Universe from gravity ( $\Phi > 0$ ) to anti-gravity ( $\Phi < 0$ ), or vice-versa which are possible for width class of cosmological potentials, but excluded for other potentials.

2) In spirit of Max principle Newton constant depends on presence of matter:  $G \sim 1/\bar{\Phi} \sim 1/\bar{\epsilon}$ .

3) For simple functions  $a(\tau)$  the cosmological factor  $\Pi(\Phi)$  and potentials  $V(\Phi)$  and  $U(\Phi)$  may show unexpected *catastrophic behavior*:  $\sim (\Delta\Phi)^{3/2}$  ( $\Delta\Phi = \Phi - \Phi(\lambda^*)$ ) in vicinity of the critical points  $\lambda^*$ :  $\Phi'(\lambda^*) = 0$  of the projection of analytical curve  $\{\Pi(\lambda), \Phi(\lambda), \lambda\}$  on the plain  $\{\Pi, \Phi\}$ . Scale factors  $a(\tau)$  yielding an *everywhere analytical cosmological factor*  $\Pi(\Phi)$  exist, too.

4) Clearly one can construct MDG model of Universe *without initial singularities*:  $a(\tau_0) = 0$  (which are typical for GR) and with *any desired kind of inflation*.

5) Because the dilaton field  $\Phi$  is quite massive, in it will be stored significant amount of energy. An interesting open question is: may the field  $\Phi$  play the role of dark matter in the Universe?

A very important problem is to reconstruct the cosmological factor  $\Pi(\Phi)$  of real Universe using proper experimental data and astrophysical observations. This problem was at first studied in [30] for more complicated models than MDG with two independent unknown functions.

We see that MDG is a rich model with new curious features and deserves further careful investigation.

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